On the measurement of $B(E2, 0_1^+ \rightarrow 2_1^+)$ using intermediate-energy Coulomb excitation

F. Delaunay[1,2] **and F. M. Nunes**[2,3]

[1] Laboratoire de Physique Corpusculaire, ENSICAEN, Université de Caen, CNRS/IN2P3, 14050 Caen, France

[2] National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824, USA

[3] Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA

E-mail: `delaunay@lpccaen.in2p3.fr`

Abstract. Coulomb excitation is a standard method used to extract quadrupole excitation strengths of even-even nuclei. In typical analyses the reaction is assumed to be one-step, Coulomb only, and is treated within a semi-classical model. In this work, fully-quantal coupled-channel calculations are performed for three test cases in order to determine the importance of multi-step effects, nuclear contributions, feeding from other states and corrections to the semi-classical approximation. We study the excitation of ^{30}S, ^{58}Ni and ^{78}Kr on ^{197}Au at ≈ 50 AMeV. We find that nuclear effects may contribute more than 10% and that feeding contributions can be larger than 15%. These corrections do not alter significantly the published $B(E2)$ values, however an additional theoretical error of up to 13% should be added to the experimental uncertainty if the semi-classical model is used. This theoretical error is reduced to less than 7% when performing a quantal coupled-channel analysis.

PACS numbers: 25.70.De, 24.10.Eq, 23.20.-g

Submitted to: *J. Phys. G: Nucl. Phys.*

Nuclear collectivity of an even-even nucleus is closely related to its quadrupole electric reduced transition probability $B(E2, 0_1^+ \rightarrow 2_1^+)$. This strength can be determined experimentally by measuring either the lifetime or the Coulomb excitation cross section of the 2_1^+ excited state [1, 2]. Originally, the Coulomb excitation technique (referred to as "Coulex" in the following) was used to measure properties of the target (e.g. [3, 4]). The sub-Coulomb energies at which the reaction took place ensured a nuclear-free measurement. In the last decade, Coulex has been expanded to intermediate energies with the aim of studying unstable nuclei [2]. In this case the nucleus of interest is the beam particle and a heavy target is used to produce the virtual photons. The reaction takes place at high enough energy to inhibit multi-step effects and data is taken only at very forward angles, where one expects to be free from nuclear interference. This method has enabled accurate measurement of the $B(E2)$ of a large variety of systems [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

A systematic comparison between the intermediate-energy Coulex method and the lifetime method showed that there is consistency between the two techniques [1]. Moreover, the accuracy of the $B(E2)$ strengths extracted through Coulex is comparable to that from the lifetime measurements [1]. The work of Cook *et al.* [1] focused on the experimental accuracy but did not consider the uncertainties due to the approximations in the Coulex theory used to connect cross sections and electric strengths. This is exactly the focus of the present study.

One way to analyse these unstable beam Coulex experiments is using the semi-classical model of Alder and Winther [18]. It conveniently provides a linear relation between the Coulex cross section and the reduced transition probability. The approximations in the Alder and Winther theory [18] are three fold: 1) the straight-line semi-classical approximation; 2) the excitation is a one-step process; 3) it is purely Coulomb. The straight-line approximation is partially corrected within Alder and Winther [18]. The second point is not so straight-forward. Most of the nuclei studied through this technique exhibit large collectivity and thus have other excited states that are strongly coupled to either the ground state or the first excited 2^+ state. Even if it is generally assumed that the cross sections to these other excited states are small at intermediate beam energies, multi-step mechanisms and interferences can distort the desired result. In order to solidify the reliability of the intermediate-energy Coulex method, it is important to evaluate the uncertainties coming from the one-step approximation. Finally, nuclear contributions need to be consistently included in the calculations so that Coulomb-nuclear interference is correctly accounted for. The inclusion of nuclear effects in realistic quantum coupled-channel calculations may enhance multi-step effects.

As mentioned above, intermediate-energy Coulex relies on restricting the scattering angles taken into account for integrating the cross sections to a range corresponding to impact parameters larger than the sum of the target and projectile radii. A detailed study of the sensitivity to the impact parameter cut was performed on the ^{46}Ar data [10] and results validate the procedure. In cases where low statistics forces the inclusion

of a wider angular range, nuclear effects have been estimated with quantum distorted wave calculations to be of the order of 6% [11, 15]. However, this value should not be taken as definitive, since, as we shall see, the nuclear contribution depends strongly on the particular analysis considered.

Another problem that is considered when analyzing Coulex data is the possibility of feeding: the reaction process excites high lying states that could then decay to the 2_1^+ state, producing an enhanced $2_1^+ \rightarrow 0_1^+$ signal. In most of the studies, estimates of feeding predict it to be unimportant (e.g. [13, 15]) mostly because at intermediate energies the relative cross sections to higher spin states are small and larger excited states are hindered compared to the lower transitions. Nevertheless there have been cases where feeding needs to be carefully considered before a reliable strength is extracted [12, 8]. In intermediate-energy Coulex experiments, the statistics is often low and the efficiency of the γ-ray detectors is limited such that a γ peak for a feeding transition is rarely seen [8]. Feeding corrections are based on theoretical estimates [18] and subtracted from the 2_1^+ cross section, before extracting the $B(E2)$ strength.

Intermediate-energy Coulex has been applied mostly to intermediate mass nuclei bound by a few MeV but as beam intensities improve, it will be applied to more exotic systems. The loosely bound nature of unstable nuclei has modified many of the traditional views of nuclear reactions. For example, when the exotic nucleus has an extended tail in its wave-functions, one finds nuclear contributions at impact parameters much larger than the sum of the target and projectile radii [19]. In addition, due to the proximity to the continuum, multi-step breakup effects need to be considered [20]. A systematic study of nuclear interference in the Coulomb dissociation of halo nuclei has shown large nuclear effects, even in the forward angular regions considered safe for Coulomb experiments [21]. A recent comprehensive study of the Coulex of ^{11}Be for extraction of the $B(E1)$ between the two bound states validates the Coulex method across a wide range of beam energies, provided all these effects are taken into account in the theoretical model [22]. For the $B(E2)$ of intermediate mass nuclei, it is important to solidify the theoretical methods used at present before these new dripline challenges can be faced.

Table 1. Information on the intermediate-energy Coulex experiments considered here. For each case we give the beam laboratory energy, the maximum centre-of-mass angle for cross-section integration, the corresponding cross section for the 2_1^+ state and the $B(E2, 0_1^+ \rightarrow 2_1^+)$ value extracted through Winther and Alder's theory [18].

Nucleus	Energy (AMeV)	θ_{CM}^{max} (deg.)	$\sigma_{2_1^+}$ (mb)	$B(E2, 0_1^+ \rightarrow 2_1^+)$ (e^2fm^4)
^{30}S	35.7	4.56	39.6(3.8)	350(33)
^{58}Ni	72.4	4.26	175(36)	707(145)
^{78}Kr	57.4	4.24	1124(133)	6244(738)

Table 2. Spin, parity and excitation energy for all the states included in the coupled-channel calculations. The 0^+ assignment for the 3.666 MeV state in ^{30}S is based on a comparison of the experimental spectrum with the spectrum of the mirror nucleus and a shell-model calculation for the $A = 30$ isobars.

^{30}S		^{58}Ni		^{78}Kr	
E (MeV)	J^π_n	E (MeV)	J^π_n	E (MeV)	J^π_n
0	0^+_1	0	0^+_1	0	0^+_1
2.211	2^+_1	1.454	2^+_1	0.455	2^+_1
3.403	2^+_2	2.459	4^+_1	1.017	0^+_2
3.666	(0^+_2)	2.775	2^+_2	1.119	4^+_1
		3.038	2^+_3	1.148	2^+_2
		3.263	2^+_4		

In this work we perform fully-quantum coupled-channel calculations for three test cases that have been measured by intermediate-energy Coulex: ^{30}S, ^{58}Ni and ^{78}Kr. These three test cases span a variety of physical situations. The first, ^{30}S, corresponds to a very short lived isotope, two nucleons away from the proton dripline, with only a few excited states. The Coulex of ^{30}S was measured at 35.7 AMeV on ^{197}Au [9]. The second, ^{58}Ni is less exotic, contains very strong transitions to higher energy states and therefore has an important feeding correction. It has been measured several times before and we consider here the experiment at 72.4 AMeV on ^{197}Au [12]. The third, ^{78}Kr, has a very small 2^+_1 excitation energy, and consequently a very large $B(E2)$. Here we consider a recent measurement at 57.4 AMeV on a ^{197}Au target [14]. Experimental details for these experiments are summarized in Table 1, where we include the $B(E2)$ extracted in the corresponding studies using the first order semi-classical theory [18].

We have investigated the spectra of these nuclei in detail and isolated the states that can affect the reaction mechanism. These are summarized in Table 2. For the two heavier cases, the spectra are well known. However, for ^{30}S, the spin and parity of the 3.666 MeV state are undetermined. We have assumed they are 0^+ by comparison with the level scheme of the ^{30}Si mirror nucleus [23] and a shell-model calculation for the $T = 1$ states of the $A = 30$ isobars [24]. As it can feed into the 2^+_1 state, it needs to be included in the calculations.

States with unnatural parity (e.g. 1^+ and 3^+ states) were not included since they would decay to 2^+ and 0^+ states by magnetic transitions which are not implemented in our coupled-channel calculations. In the energy range of interest, there is one 1^+ state in ^{30}S, one 1^+ state in ^{58}Ni and none in ^{78}Kr. Generally, magnetic transitions in Coulex are much weaker than the electric ones [25].

A very large amount of transitions is possible between the excited states listed in Table 2. In Table 3 we list all the transitions taken into account in our coupled-channel calculations. We also provide halflives and branching ratios [23] from which we

Table 3. Transitions included in the calculations: spins and parities of the states involved in each transition, the corresponding halflives $T_{1/2}$, branching ratios I_γ, the reduced transition matrix elements $M(E\lambda)$ and deformation lengths $\delta = \beta R$. Unless otherwise noted, the spins, parities, halflives and branching ratios were taken from the NNDC database [23].

	$J^\pi_{n,1}$	$J^\pi_{n,2}$	$T_{1/2}$	I_γ	$M(E\lambda)$ ($e$fm$^\lambda$)	βR (fm)
^{30}S	2^+_1	0^+_1		1.00	18.71 [a]	1.314 [a]
	2^+_2	0^+_1	115 fs	0.20	3.29	0.231
	2^+_2	2^+_1	115 fs	0.80	90.51	2.842
	0^+_2	2^+_1	1 ps [b]	1.00	9.31	0.292
^{58}Ni	2^+_1	0^+_1		1.00	26.59 [a]	0.856 [a]
	4^+_1	0^+_1	970 fs [b]	2×10^{-8} [c]	481.08	0.718
	4^+_1	2^+_1	970 fs [b]	1.00	71.56	1.031
	2^+_2	0^+_1	0.38 ps	4.3×10^{-2}	1.40	0.045
	2^+_2	2^+_1	0.38 ps	0.96	42.13	0.607
	2^+_2	4^+_1	0.38 ps	5.7×10^{-4}	36.75	0.395
	2^+_3	0^+_1	52 fs	0.40	9.18	0.296
	2^+_3	2^+_1	52 fs	0.58	56.58	0.815
	2^+_3	4^+_1	52 fs	2.9×10^{-3}	49.62	0.533
	2^+_3	2^+_2	52 fs	9.9×10^{-3}	72.18 [d]	0.775 [d]
	2^+_4	0^+_1	35 fs	0.59	11.41	0.367
	2^+_4	2^+_1	35 fs	0.39	40.45	0.583
	2^+_4	4^+_1	35 fs	1.0×10^{-2}	49.35	0.530
	2^+_4	2^+_2	35 fs	1.8×10^{-3}	72.18	1.040
^{78}Kr	2^+_1	0^+_1		1.00	79.02 [a]	1.793 [a]
	0^+_2	2^+_1	5.8 ps [e]	1.00	41.89 [e]	0.425 [e]
	4^+_1	2^+_1	2.5 ps	1.00	125.68	1.275
	2^+_2	0^+_1	3.7 ps	0.39	12.21	0.277
	2^+_2	2^+_1	3.7 ps	0.61	54.29	0.551

[a] using $B(E2, 0^+_1 \rightarrow 2^+_1)$ measured by Coulex [9, 12, 14]
[b] experimental lower value
[c] I_γ of the $4^+_1 \rightarrow 0^+_1$ transition in ^{60}Ni
[d] from the $2^+_4 \rightarrow 2^+_2$ transition in ^{58}Ni
[e] assuming $B(E2, 0^+_2 \rightarrow 2^+_1) = B(E2, 4^+_1 \rightarrow 2^+_1)$

determined the $B(E\lambda)$. Reduced matrix elements $M(E\lambda)$ were evaluated directly from $B(E\lambda)$:

$$M(E\lambda, I_1 \rightarrow I_2) = \sqrt{(2I_1+1)B(E\lambda, I_1 \rightarrow I_2)}.$$

For the $0^+_1 \rightarrow 2^+_1$ transitions, we used $B(E2)$ directly from the Coulex experiments

Table 4. One-step (DWBA) and coupled-channel (CC) calculations including only the ground state and the 2_1^+ state: comparison between cross sections integrated over the experimental angular range for the full process with those from Coulomb only. All cross sections are in mb.

	DWBA(2_1^+)	CC(2_1^+)
^{30}S Coul+nucl	44.7	43.6
^{30}S Coulomb	40.6	40.6
^{58}Ni Coul+nucl	161.5	161.1
^{58}Ni Coulomb	183.8	182.7
^{78}Kr Coul+nucl	1052.6	1030.7
^{78}Kr Coulomb	1056.3	1036.0

data under scrutiny [9, 12, 14]. We have not included transitions with reduced matrix elements $M(E\lambda)$ two orders of magnitude lower than the main $0_1^+ \rightarrow 2_1^+$ transition. Since our coupled-channel calculations are restricted to electric couplings, transitions involving a mixture of $M1$ and $E2$ transitions (such as $2^+ \rightarrow 2^+$ transitions) were assumed to be 100% $E2$. This is not always a good approximation. However, none of these transitions is involved in the direct excitation of the 2_1^+ states, but in feeding couplings. Our aim was to study the effects of couplings, therefore we chose to keep the full strength of these transitions. In our calculations we assumed all coupling amplitudes have the same sign. We have checked this assumption and verified that uncertainties due to sign changes are negligible.

A specific remark is needed for ^{78}Kr, as the lifetime of the 0_2^+ state is unknown. This state, as well as the 4_1^+ and 2_2^+ states, lies at about twice the excitation energy of the 2_1^+ state. This suggests that these 0_2^+, 2_2^+ and 4_1^+ states are quadrupole 2-phonon states, with the 2_1^+ being the 1-phonon state. In a pure vibrational model, $B(E2, 0_2^+ \rightarrow 2_1^+) = B(E2, 4_1^+ \rightarrow 2_1^+)$ [26]. This is what we assumed in order to obtain the transition matrix element for the $0_2^+ \rightarrow 2_1^+$ transition in ^{78}Kr.

As part of our aim is to check the importance of nuclear effects, nuclear couplings were also included in the calculations. We have assumed the matter deformation to be the same as the charge deformation for all three cases. As these systems are not halo-like, this approximation should be adequate. The corresponding deformation lengths are also included in Table 3. For a given transition, the nuclear and Coulomb couplings were considered in both directions, but reorientation couplings were not included, as would be the case in a vibrational model. Optical model parameters were taken from elastic scattering studies. For the ^{30}S case, we used the optical model parameters for ^{40}Ar + ^{208}Pb at 44 AMeV [27]. For ^{58}Ni and ^{78}Kr on ^{197}Au, we used the parameters from an elastic scattering study of ^{86}Kr + ^{208}Pb at 43 AMeV [28]. A value of $r_C = 1.2$ fm was used for the Coulomb radius parameter. Coupled-channel calculations were performed using the code FRESCO [29].

Table 5. Full coupled-channel calculations including all transitions specified in Table 3. $I_{J_n^\pi \to 2_1^+}$ is the branching ratio for the $J_n^\pi \to 2_1^+$ transition [23]. σ_{feed} are the cross sections for feeding into the 2_1^+ state. The value for σ_{feed} in bold is the sum of the cross sections contributing to $\sigma(2_1^+)$ for each case.

Nucleus	State	Full CC		
		σ (mb)	$I_{J_n^\pi \to 2_1^+}$	σ_{feed} (mb)
^{30}S	2_1^+	43.6		**45.5**
	2_2^+	2.2	0.80	1.8
	0_2^+	0.1	1.00	0.1
^{58}Ni	2_1^+	155.6		**188.3**
	4_1^+	8.6	1.00	8.6
	2_2^+	1.9	0.96	1.8
	2_3^+	19.1	0.58	11.1
	2_4^+	29.0	0.39	11.3
^{78}Kr	2_1^+	1013.9		**1041.9**
	0_2^+	2.8	1.00	2.8
	4_1^+	8.0	1.00	8.0
	2_2^+	28.1	0.61	17.2

For an accurate calculation of the Coulex cross section at forward angles one needs to be very careful with convergence. Partial waves up to $L_{max} = 3000-6000$ and a radial integration up to $R_{max} = 300$ fm were needed in order to get a converged integrated cross section within the experimental angular range. Checks of the sensitivity of the 2_1^+ excitation cross section to the optical model parameters were performed and we estimate an uncertainty smaller than 5%. Our calculations show negligible sensitivity to the Coulomb radius r_C. It is important to note that the calculations we have performed are non-relativistic. Relativistic kinematical effects have been studied within the context of the first-order semi-classical theory [30]. We have estimated the effect of relativistic kinematics by repeating the calculations at a corrected beam energy and obtained for all three cases modifications in the cross sections smaller than 5%. Therefore, if we neglect uncertainties in the transition strengths used, the theoretical error on our cross sections should be smaller than 7%. This should be added to the experimental errors.

Results including only the ground and 2_1^+ states are presented in Table 4. We present 1-step (DWBA) and coupled-channel (CC) calculations. Cross sections were integrated over the angular range of the particular experiment. We performed calculations including both nuclear and Coulomb excitations in the transition matrix elements and we compare them with the results obtained with only Coulomb. In all three cases, $0_1^+ \leftrightarrow 2_1^+$ multi-step effects are small. Nuclear effects are of the order of 10%, 12% and 0.4% for ^{30}S, ^{58}Ni and ^{78}Kr, respectively. These are still within the

experimental limits.

For ^{30}S and ^{58}Ni, nuclear effects are not negligible and indicate that the maximum angle used in integrating the experimental cross section should be carefully chosen. The contribution of the nuclear part of the interaction to the cross section is indeed very sensitive to this maximum angle. For ^{58}Ni, our tests show that decreasing the maximum centre-of-mass angle by only 0.5 degree cuts the relative nuclear contribution by a factor 2.

One can also compare our Coulomb-only DWBA cross sections to those predicted by the semi-classical model (see Table 1). From this comparison we find that the straight-line trajectory approximation alone introduces an error in the cross section of 6% at the most.

Full coupled-channel results including the states in Table 2 and transitions from Table 3 are presented in Table 5. The cross sections to individual states (σ) are multiplied by the branching ratios to the 2_1^+ state in order to get the feeding contributions. The sum of these and the Coulex 2_1^+ cross section gives the full cross section (in bold) to be compared to the experiment. Feeding contributions are important in ^{58}Ni ($\approx$ 17%), as we already knew, but also in ^{30}S ($\approx$ 5%). For ^{58}Ni, the feeding correction estimated in [12] of 25 mb is significantly smaller than our prediction (33 mb), mainly because of the contribution from the 4_1^+ state which was omitted in the experimental estimation [12]. For ^{30}S our theoretical cross section taking $B(E2, 0_1^+ \rightarrow 2_1^+) = 350$ e^{2}fm^2 is above the upper limit of the experimental range. A reduction of the $B(E2)$ value to 303 e^{2}fm^4 is necessary to get a theoretical cross section in agreement with the experimental result. Considering the 7% theoretical error, this value is still compatible with the value extracted in [9]. For ^{58}Ni and ^{78}Kr, the expected cross section is within $\approx$ 7% of the mean experimental value, i.e. within the experimental range.

In conclusion, full coupled-channel calculations for ^{30}S, ^{58}Ni and ^{78}Kr Coulex confirm the agreement between $B(E2)$ extracted through the lifetime and the Coulex methods [1]. Our study shows that theoretical contributions to the errors in the cross sections can be $\approx$ 13% if the first order semi-classical theory of Alder and Winther is used [18] but should be less than 7% if a full quantum coupled-channel calculation is performed and feeding is consistently taken into account.

We thank Alexandra Gade for useful discussions and comments on earlier versions of the manuscript. This work was supported by NSCL, Michigan State University, and the National Science Foundation through grant PHY-0555893.

References

[1] Cook J M, Glasmacher T and Gade A 2006 *Phys. Rev.* C **73** 024315
[2] Glasmacher T 1998 *Annu. Rev. Nucl. Part. Sci* **48** 1
[3] Shaw A H and Greenberg J S 1974 *Phys. Rev.* C **10** 263
[4] Ahmad A *et al.* 1988 *Phys. Rev.* C **37** 1836
[5] Motobayashi T *et al.* 1995 *Phys. Lett.* B **346** 9

[6] Scheit H *et al.* 1996 *Phys. Rev. Lett.* **77** 3967
[7] Ibbotson R W *et al.* 1998 *Phys. Rev. Lett.* **80** 002081
[8] Cottle P D *et al.* 1999 *Phys. Rev.* C **60** 031301(R)
[9] Cottle P D *et al.* 2002 *Phys. Rev. Lett.* **88** 172502
[10] Gade A *et al.* 2003 *Phys. Rev.* C **68** 014302
[11] Yurkewicz K L *et al.* 2004 *Phys. Rev.* C **70** 034301
[12] Yurkewicz K L *et al.* 2004 *Phys. Rev.* C **70** 054319
[13] Dinca D C *et al.* 2005 *Phys. Rev.* C **71** 041302(R)
[14] Gade A *et al.* 2005 *Phys. Rev. Lett.* **95** 022502
[15] Church J A *et al.* 2005 *Phys. Rev.* C **72** 054320
[16] Banu A *et al.* 2005 *Phys. Rev.* C **72** 061305(R)
[17] Perru O *et al.* 2006 *Phys. Rev. Lett.* **96** 232501
[18] Winther A and Alder K 1979 *Nucl. Phys.* A **319** 518
[19] Nunes F M and Thompson I J 1998 *Phys. Rev.* C **57** R2818
[20] Nunes F M and Thompson I J 1999 *Phys. Rev.* C **59** 2652
[21] Hussein M S, Lichtenthäler R, Nunes F M and Thompson I J 2006 *Phys. Lett.* B **640** 91
[22] Summers N C *et al.* 2007 *Phys. Lett.* B *in press*
[23] http://www.nndc.bnl.gov/
[24] http://www.nscl.msu.edu/∼brown/resources/SDE.HTM
[25] Alder K *et al.* 1956 *Rev. Mod. Phys.* **28** 432
[26] Bohr A and Mottelson B 1975 *Nuclear Structure* vol 2 *Nuclear Deformations* (Reading: Benjamin)
[27] Alamanos N *et al.* 1984 *Phys. Lett.* B **137** 37
[28] Roussel-Chomaz P *et al.* 1988 *Phys. Lett.* B **209** 187
[29] Thompson I J 1988 *Compt. Phys. Rep.* **7** 3
[30] Bertulani C A, Stuchbery A E, Mertzimekis T J and Davies A D 2003 *Phys. Rev.* C **68** 044609